Self-sustained plasma waveguide structures produced by ionizing laser radiation in a dense gas

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The propagation of high-power laser radiation producing rapid ionization in a dense gas is analyzed by using a nonlinear electrodynamical model. It is shown that the interplay between the Kerr-type and the defocusing ionization nonlinearities may lead to the formation of self-sustained plasma waveguide filaments. Quasi-steady-state laser-plasma structures supported by various electromagnetic field configurations are considered.

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One of the most striking achievements of laser physics in recent years is the development of methods for generation of ultrashort high-power laser pulses. This has intensified the interest in investigating various aspects of the interaction of laser radiation with matter. In particular, the possibility to generate long plasma channels in a dense gas by means of high-intensity laser radiation, and the propagation of the laser light through these plasma channels is a problem that has important implications for xuv laser developments and laser-driven particle acceleration [1]. It may also play a fundamental role in the process of significant frequency spectrum transformation of intense laser pulses that are focused into dense gases [1–3].

In this paper, we examine the effect of laser pulse channeling due to the creation of self-sustained plasma-field structures in the presence of strong ionization of a dense gas with a Kerr-type atomic nonlinearity. It seems to be quite paradoxical that the ionization that usually has a strong defocusing effect on the radiation in the created plasma region [4,5] may be the origin of waveguiding. Nevertheless, recent experimental results [6,20–22], where short-pulse self-channeling has been observed over distances of several Rayleigh lengths, showed that the presence of a strong ionization does not destroy the guiding effect. The aim of this paper is to demonstrate that the interplay between the Kerr-type focusing nonlinearity and the threshold-type ionization nonlinearity may result in the formation of steady-state plasma-field structures where the guiding along the created plasma core is attained owing to the gas ionization.

The combined effect of focusing and defocusing nonlinearities (heating and ionization, Kerr effect, and ionization) has been studied theoretically in [7,8]. However, the attention in Refs. [7,8] has been paid to the competition between these nonlinearities rather than to their coexistence. In particular, Liu and Umstadter [8] have investigated a multiple-peak field structure that emerged for the tight focusing of an ionizing terawatt laser pulse and the refractive role of the ionization limiting the field amplitude. Here, we shall analyze the opportunity to create a quasi-steady-state waveguide, aiming at an increase of the laser-plasma interaction length for the above-mentioned applications.

We start the analysis by first considering ionization processes that occur in dense gases in the presence of laser radiation. The evolution of the plasma electron density $N(\mathbf{r},t)$ is described by the ionization balance equation

$$\partial N/\partial t = \gamma(N, N_m, |\mathbf{E}|) , \tag{1}$$

where the ionization rate γ is, in general, a function of the electron density, the neutral gas density N_m, and the amplitude of the electromagnetic field $|\mathbf{E}|$. We have omitted the dependence of γ on the phase of the field, which may be essential for the energy balance in the case of linearly polarized radiation [9,10] but which is not important for the spatial field structure. Our analysis neglects other mechanisms of electron density change, e.g., ponderomotive and thermal effects that are usually investigated in the case of already created fully ionized plasmas. During a short laser pulse (<1 ps), these nonlinearities cannot significantly affect the refractive index in comparison with the rapid ionization that enhances the electron density by many orders of magnitude. In the following we will discuss the cases of optical field-induced ionization and electron-impact ionization separately. The field-induced ionization plays a dominant role in the case of rather short electromagnetic pulses when the density of free electrons does not increase, during the pulse time, to a level at which the collisional processes become important. The ionization rate function is characterized by a sharp dependence on the field intensity, which is experimentally treated as the existence of an ionization threshold. Theoretically, the rate of multiphoton or tunneling ionization γ is usually approximated by a power-like or an exponentially growing function of $|\mathbf{E}|$ [11]. In order to be able to describe the plasma structure analytically we will adopt the following simplified model for the function γ:

$$\gamma(N, N_m, |\mathbf{E}|) = \begin{cases} \infty , & |\mathbf{E}| > E_{\text{th}} \\ 0 , & |\mathbf{E}| \le E_{\text{th}} , \end{cases} \tag{2}$$

where E_{th} is the field threshold value for ionization, which depends on the sort of gas and on the laser radiation parameters such as carrier frequency and field polarization. The infinite value of the ionization rate in Eq.

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(2) means that, for a field exceeding locally the threshold value, the electron density immediately becomes high enough (with respect to the pulse duration) to stop a further field growth due to the refraction. We assume also that the gas pressure is sufficient to provide the required number of free electrons. Thus, in the model of instantaneous threshold-type ionization the electric field turns out to be self-limited at the threshold value inside the plasma region. Recent computer experiments with more sophisticated ionization models [5,12] have demonstrated similar effects of self-limitation. Thus we conclude that the simplified ionization model given by Eq. (2) should be qualitatively correct although the quantitative accuracy is more uncertain. Formally, Eqs. (1) and (2) imply that, in the presence of the laser pulse and under the condition of self-limitation, a quasisteady state of the plasma can be described by

$$N=\begin{cases}N(\mathbf{r}) & \text{at } |\mathbf{E}|=E_{\text{th}} ,\\ 0 , & |\mathbf{E}|<E_{\text{th}} .\end{cases} \tag{3}$$

This expression is obviously valid for the bulk of the pulse and does not hold near the front where the plasma density remains nonzero in the region of evanescent field. Note that the model for the plasma distribution (3) is similar to that used in investigations of long-pulse microwave discharges [13,14] where the electron-impact ionization is a dominant process and where charged particle losses have to be taken into account.

If the main loss mechanism is the attachment of free electrons to electronegative molecules we have

$$\gamma=\{\nu_i(|\mathbf{E}|)-\nu_a\}N , \tag{4}$$

where ν_i is the impact ionization frequency, ν_a is the attachment frequency, which is usually a very slowly varying function of $|\mathbf{E}|$ as compared to $\nu_i(|\mathbf{E}|)$. In steady state Eqs. (1) and (4) yield exactly the expression (3) where the threshold field E_{th} is determined by $\nu_i(|\mathbf{E}|=E_{\text{th}})=\nu_a$.

The neutral component of the gas is responsible for the Kerr effect, which arises due to the nonlinear motion of bound electrons in the upper atomic shells. The bound electron nonlinear response is local in time (at least for the available laser pulse durations >10 fs) and follows the intensity of the optical field. An atom subjected to ionization usually does not contribute to the Kerr effect because of the essential difference between the atomic and ionic nonlinear polarizations. We consider rather dense gases where a small ionization degree is sufficient to prevent the field growth due to the Kerr-effect self-focusing. Because the bulk of the atoms remain neutral, the characteristic Kerr nonlinearity field can be taken as being independent of the free electron density. At the same time, the contribution of the ionization nonlinearity to the change of the gas refractive index is significant even at small ionization degrees due to a strong polarization response of detached electrons as compared to the bound ones. Hence, we have two nonlinearities of different signs and of different dependences on the laser pulse intensity.

Let us now proceed by deducing the spatial structure of the electromagnetic field in the emerging plasma. Our aim is to demonstrate the guiding effect, keeping in mind that in the reference frame connected with the propagating laser pulse the field distribution is quasistationary and localized in the transverse direction. Note that the very leading low-intensity part of the pulse, which obeys the linear diffraction law, cannot be guided and therefore it is excluded from the consideration. The conditions that facilitate the implementation of the guiding effect are a weak radiation focusing and a large beam aperture geometry. Considering the propagation of an s-polarized electromagnetic wave, we use the nonlinear Helmholtz equation to describe the evolution of the slowly varying amplitude of the electric field $E(\mathbf{r})\exp(i\omega t)$. Thus

$$\nabla^2 E+(\omega^2/c^2)\{1-(N/N_c)[1+i(\nu/\omega)]+\beta|\mathbf{E}|^2\}\mathbf{E}=0 , \tag{5}$$

where ω is the frequency of the laser radiation, ν is the electron-atom collision frequency, $N_c=m(\omega^2+\nu^2)/4\pi e^2$ is the critical electron density, and β is the coefficient of the Kerr nonlinearity. Since $\beta=4\pi e^2 N_m/m\omega_a^2 E_a^2$, where $\omega_a\cong I_a/\hbar$, m is the electron mass, e is the electron charge, $\hbar$ is Planck's constant, I_a is the ionization potential of the atom, and E_a is the "atomic" electric field, the self-focusing effect is of importance at large values of the gas density N_m. When $\beta=0$ we note that Eqs. (3) and (5) are equivalent to the model used in microwave discharge theory to describe stationary plasma structures supported by external fields [14].

The equation system (3) and (5) describes consistently the following electromagnetical problem: with a given source of electromagnetic radiation we have to find an electron density distribution such that inside the plasma region, where $N(\mathbf{r})\neq 0$, the electric field amplitude is equal to the threshold value for ionization; outside the plasma, where $N(\mathbf{r})=0$, the field amplitude is below the threshold.

We express the electric field as $\mathbf{E}=\mathbf{A}\exp(i\varphi)$ and separate the real and imaginary parts of Eq. (5) to obtain

$$\nabla\cdot(|\mathbf{A}|^2\nabla\varphi)=\delta n|\mathbf{A}|^2 , \tag{6}$$

$$\nabla^2\mathbf{A}-(\nabla\varphi)^2\mathbf{A}+(1-n+\beta|\mathbf{A}|^2)\mathbf{A}=\mathbf{0} , \tag{7}$$

where we have introduced the dimensionless variables $\mathbf{r}\omega/c\to\mathbf{r}$, $N/N_c=n$, and $\nu/\omega=\delta$. Substituting Eq. (3) into Eqs. (6) and (7) yields the following relations: (i) Inside the plasma, region where $|\mathbf{A}|$ is equal to the constant E_{th}, we have

$$\nabla^2\varphi=\delta n , \tag{8}$$

$$(\nabla\varphi)^2=1-n+\beta E_{\text{th}}^2 . \tag{9}$$

(ii) Outside the plasma region, where $\mathbf{n}=0$ and $|\mathbf{A}|<E_{\text{th}}$, one obtains

$$\nabla\cdot(|\mathbf{A}|^2\nabla\varphi)=0 , \tag{10}$$

$$\nabla^2\mathbf{A}+[1-(\nabla\varphi)^2+\beta|\mathbf{A}|^2]\mathbf{A}=\mathbf{0} . \tag{11}$$

At the plasma boundary the continuity conditions for the field amplitude and its derivative must be satisfied.

The equation system (8)–(11) together with the boundary conditions describe all stationary plasma structures that can be produced by powerful laser radiation in a dense gas. However, to obtain analytical results that clearly demonstrate the physical significance of the problem we will restrict the analysis by considering only two specific field configurations. Focusing our attention on a two-dimensional model in which the light field propagates along the z axis, we look for solutions of Eqs. (8)–(11) in the form $\mathbf{A}=\hat{\mathbf{y}}A(x)$, $\varphi=-kz+\Phi(x)$, and $n=n(x)$, where k is the wave number along the direction of propagation.

Inside the plasma region Eqs. (8) and (9) then reduce to

$$(d/dx)\sqrt{n_m-n}=\delta n\ , \tag{12}$$

where $n_m=1+\beta E_{\rm th}^2-k^2$. Integrating (12), we obtain

$$n(x)=\begin{cases} n_m\,\mathrm{sech}^2(\delta\sqrt{n_m}x)\ , & |x|\le d \\ 0\ , & |x|>d\ , \end{cases} \tag{13}$$

where d is the plasma transverse dimension, normalized by c/ω. Outside the plasma Eq. (10) yields $A^2(d\Phi/dx)=J=\mathrm{const}$; i.e., the transverse power flux density is nonzero due to the plasma absorption. Substituting this into Eq. (11), we find

$$(d^2A/dx^2)+(1-k^2+\beta A^2)A-(J^2/A^3)=0\ . \tag{14}$$

Let us first analyze a simplified but instructive case in which absorption effects can be neglected, i.e., $\delta=\nu/\omega=0$. Then Eq. (13) yields $n(x)=n_m=\mathrm{const}$ for $|x|\le d$, which describes a homogeneous plasma channel. In the region outside the plasma, it follows from Eqs. (9) and (10) that $J=0$ and the localized solution of Eq. (14) is given by $A(x)=[(2/\beta)(k^2-1)]^{1/2}\mathrm{sech}[(k^2-1)^{1/2}(x-d)]$, $|x|\ge d$, where $k>1$. Applying the continuity conditions at $x=d$, we find that $A(x=d)=E_{\rm th}$, which gives $k=(1+\beta E_{\rm th}^2/2)^{1/2}$. Thus, the self-sustained filament has the following structure:

$$n(x)=\begin{cases} \tfrac{1}{2}\beta E_{\rm th}^2\ , & |x|\le d \\ 0\ , & |x|>d\ , \end{cases} \tag{15a}$$

$$A(x)=\begin{cases} E_{\rm th}\ , & |x|\le d \\ E_{\rm th}\mathrm{sech}[\sqrt{\beta/2}E_{\rm th}(x-d)]\ , & |x|>d\ . \end{cases} \tag{15b}$$

The plasma dimension d is determined by the total incident beam power $P=\int_{-\infty}^{+\infty}A^2\,dx$. It follows from (16b) that $d=(P-P_{\rm th})/2E_{\rm th}^2$, where $P_{\rm th}=2\sqrt{2/\beta}E_{\rm th}$ or in physical parameters $P_{\rm th}=(m\omega_a^2/32\pi^{3/2}e^2\omega^2)^{1/2}c^2E_aE_{\rm th}$. Thus, to create a plasma filament defined by (16) it is of course necessary to exceed the threshold field for ionization by focusing. In addition the beam power has to be larger than the threshold power value $P_{\rm th}$.

In the presence of absorption effects ($\delta\neq 0$) a longitudinally unlimited plasma filament supported by a localized wave field cannot exist. Due to absorption of laser radiation, the waveguide plasma has a limited longitudinal size, and the supporting field structure will give rise to a transverse energy flux across the plasma boundary. Since $J\neq 0$, Eq. (14) has only periodic solutions. Then, depending on the normalized wave number value, k, two cases can be distinguished. If $k<1$, the value of k can be fixed and defined by the external conditions of the problem (unlike the above analysis where k has been determined by the continuity of the field). A similar problem but without local nonlinearity ($\beta=0$) has been solved in Ref. [14]. It can be shown that the effect of $\beta\neq 0$ does not change the solutions qualitatively. If $k>1$, the local nonlinearity plays an important role as it is responsible for the existence of localized plasma structures. Using Eqs. (9) and (10) we obtain the power flux density J as

$$J=E_{\rm th}^2(d\Phi/dx)|_{x=d}=E_{\rm th}^2[1+\beta E_{\rm th}^2-k^2-n(d)]^{1/2}\ . \tag{16}$$

The solution of Eq. (14) is then given by $A^2(x)=u_2+(u_3-u_2)\times\mathrm{cn}^2\{(x-d)[(\beta/2)(u_3-u_1)]^{1/2},s\}$, where u_i $(i=1,2,3)$ are the roots of the equation $\beta u^3-2(k^2-1)u^2-4Cu+2J^2=0$, $u_1<u_2\le u_3$, the constant C is related to u_i, $s^2=(u_3-u_2)/(u_3-u_1)$, and cn denotes the Jacobian elliptic function. Thus the function $A^2(x)$ represents a cnoidal wave bounded between u_2 and u_3 and having a period equal to $\sqrt{(8/\beta)(u_3-u_1)}K(s^2)$, where $K(s^2)$ is the complete elliptic integral of the second kind. From the boundary condition at $x=d$, we find $u_3=E_{\rm th}^2$, which determines the wave number k. Note that, in experiments, the energy flow in the transverse cross-section supporting the dissipative filament can be produced by using the axicon lens focusing [15].

Let us now discuss the solutions of Eqs. (8)–(11) in cylindrical geometry where the plasma density distribution is $n=n(r)$. Strictly speaking, the s-polarization wave model is no longer valid for this symmetry. However, assuming that $n\ll 1$, which is the case for the experiments on high-intensity laser-gas interaction, we can use the well-known scalar field approximation [16]. Considering the case of a collisionless plasma ($\delta=0$) and substituting $A=A(r)$ and $\varphi=\varphi(z)$, we find from Eqs. (8) and (9) that $A=E_{\rm th}$ and the density of a homogeneous cylindrical plasma channel, $r\le d$, is given by $n=1-k^2+\beta E_{\rm th}^2$, where k is the wave number of the guided wave. Such a filament may be supported by a localized wave field, which outside the plasma region ($r>d$) obeys the equation

$$\nabla_\perp^2A+(1-k^2+\beta A^2)A=0\ . \tag{17}$$

This equation is well known in the theory of optical beam self-focusing and it has a localized solution, the so-called Townes mode for $0\le r<\infty$ and $k^2>1$ [17]. A characteristic feature of the Townes mode is that it may exist only at a critical beam power $P_{\rm cr}=2\int_0^\infty A^2r\,dr=11.8/\beta$. For the problem considered here, we can construct a similar localized solution, $A=A_s(r)$, in the interval $d\le r<\infty$, with the boundary conditions $(dA_s/dr)_{r=d}=0$ and $A_s\ (r=d)=E_{\rm th}$. Similarly to the above analysis, the latter conditions will determine the value of k and therefore the density of the plasma channel. Note that the width of the plasma channel d is defined by the incident radiation power $P=2\int_0^\infty A^2r\,dr=d^2E_{\rm th}^2+2\int_d^\infty A_s^2r\,dr$, which implies the existence of a critical power required to create the plasma channel. If the incident power exceeds the critical value, self-focusing of the laser pulse will take place at the initial stage of the evolution, and a plasma will be generated when the wave field amplitude exceeds the threshold for ionization. Then, the dynamics of the pulse is determined by an interplay between the self-focusing effect that leads to beam compression and the defocusing effect due to a nonuniform generated plasma. The final stage may be completed by formation of a self-sustained plasma filament that channels electromagnetic energy along the filament without refraction of the laser pulse. It follows from Fig. 1 that the creation of plasma filaments with large diameters requires high levels of incident laser power, essentially exceeding the critical power value. At the same time, the plasma density in the filaments varies weakly around the approximate value $N \approx N_c \beta E_{\rm th}^2 = N_m(\omega/\omega_a)^2(E_{\rm th}/E_a)^2$. In the case when $P \gg P_{\rm cr}$ it can be expected that, at the first stage, the Kerr effect induces a splitting of the field distribution into a number of secondary beams [18] having powers close to the threshold value. Then, at the second stage, each secondary beam will experience self-focusing with the subsequent plasma-core formation and guiding. The whole picture will look like a multichanneling of the laser radiation by means of gas ionization.

In conclusion, we estimate experimental conditions at which the discussed effect may be observed. For dense gases at pressures $p = 10^2 - 10^3$ Torr, the critical power of Kerr-effect induced self-focusing which sets the lower limit for guiding, is in the range of $10^2 - 10^3$ GW [19]. Therefore, an energy of about 10–100 mJ in several hundreds of femtoseconds is needed, which is typical for modern high-power facilities based on the near-infrared laser sources [Nd:YAG (yttrium aluminum garnet), Ti:Sa, etc.]. As the power requirement is fulfilled, the key question concerns the plasma-field matching condition. It can be achieved for a rather weak beam focusing, which provides a gradual filament formation in space rather than a sharp refraction from a rapidly ionized focus region. This condition is not quite typical for the high-power laser-plasma experiments where the traditional approach aims at maximum concentration of the electromagnetic energy in a tight focusing geometry. In a very recent experiment with 200 fs and with 20-mJ Ti:Sa laser radiation focused in air at a length > 1 m [20–22], the matching condition has been fulfilled. This resulted in a well-pronounced guiding effect over a 20-m distance. Let us estimate the possible guiding distance, keeping in mind the above theoretical results and the experimental parameters of Refs. [20–22]. The length of the filament, 1, is determined by the energy dissipation due to creation of the plasma. Thus, approximately, we can write $1 = W/I_a N d^2$, where W is the pulse energy, I_a is the ionization potential, and d is the plasma radius. Using the expression for the electron density in the filament given above and taking the ionization threshold $E_{\rm th}/E_a = 0.1$ [23,24], we obtain $1 = 20$ m. This result is consistent with the experimentally observed guiding distance (20 m), which unfortunately was limited by the available space of the experiment. However, the experimental papers clearly identify the Kerr-effect nonlinearity and ionization as the important physical mechanisms and we conclude that the theory developed in the present paper properly describes the effect of plasma-core optical guiding over many Raleigh lengths of ultrashort optical pulses producing gas ionization.

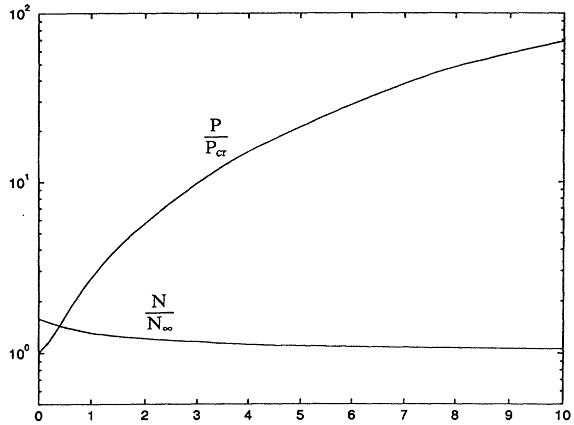

FIG. 1. The calculated laser power and plasma density as a function of filament diameter. The plasma density is normalized to the limiting value at $d \gg 1$, namely, $N_\infty = N_c \beta E_{\rm th}^2/2$.

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